Chapter 1: Real Numbers

EXERCISE 1.1

1. Express each number as a product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Solution 1:
(i) 140
Step 1: Check smallest prime factor.

140 ÷ 2 = 70
70 ÷ 2 = 35

Step 2: 35 is not divisible by 2, try 3 (no), try 5 → 35 ÷ 5 = 7
Step 3: 7 is a prime number.
So:

$$140 = 2 \times 70$$
$$= 2 \times 2 \times 35$$
$$= 2 \times 2 \times 5 \times 7$$
$$\boxed{140 = 2^2 \times 5 \times 7}$$

(ii) 156
Step 1:
156 ÷ 2 = 78
78 ÷ 2 = 39
Step 2: 39 ÷ 3 = 13
Step 3: 13 is prime.
So:

$$156 = 2 \times 78$$
$$= 2 \times 2 \times 39$$
$$= 2 \times 2 \times 3 \times 13$$
$$\boxed{156 = 2^2 \times 3 \times 13}$$

(iii) 3825
Step 1: Check divisibility:
Last digit is 5, so divisible by 5:
3825 ÷ 5 = 765
Step 2: 765 ends with 5 → divisible by 5:
765 ÷ 5 = 153
Step 3: 153 → sum of digits = 1 + 5 + 3 = 9 → divisible by 3:
153 ÷ 3 = 51
Step 4: 51 ÷ 3 = 17
Step 5: 17 is prime.
So:

$$3825 = 5 \times 765$$
$$= 5 \times 5 \times 153$$
$$= 5 \times 5 \times 3 \times 51$$
$$= 5 \times 5 \times 3 \times 3 \times 17$$
$$\boxed{3825 = 3^2 \times 5^2 \times 17}$$

(iv) 5005

Step 1: Check divisibility by 5 (ends with 5):

5005 ÷ 5 = 1001

Step 2: 1001 — check divisibility by 7 (since 7 × 143 = 1001, yes):

1001 ÷ 7 = 143

Step 3: 143 = 11 × 13

So:

$$5005 = 5 \times 1001$$
$$= 5 \times 7 \times 143$$
$$= 5 \times 7 \times 11 \times 13$$
$$\boxed{5005 = 5 \times 7 \times 11 \times 13}$$

(v) 7429

Step 1: Check small primes: not divisible by 2, 3, 5.

Try 7: 7 × 1061 = 7427 (remainder 2) → no.

Try 11: 11 × 675 = 7425 (remainder 4) → no.

But we can try 17 (since sometimes in NCERT, 7429 = 17 × 19 × 23 possibly):

Let's test 17:

17 × 437 = 7429 exactly (since 17 × 400 = 6800, 17 × 37 = 629, total 7429).

So, 7429 ÷ 17 = 437.

Step 2: 437 ÷ 19 = 23 (since 19 × 23 = 437).

Step 3: 23 is prime.

Thus:

$$7429 = 17 \times 437$$
$$= 17 \times 19 \times 23$$
$$\boxed{7429 = 17 \times 19 \times 23}$$

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2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.
(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Solution 2:
(i) 26 and 91
Step 1: Prime factorization

$$26 = 2 \times 13$$
$$91 = 7 \times 13$$

Step 2: HCF = common factor(s) = 13

Step 3: LCM = product of highest powers of all prime factors

$$LCM = 2 \times 7 \times 13 = 182$$

Step 4: Verification

$$HCF \times LCM = 13 \times 182 = 2366$$
$$26 \times 91 = 2366$$

Equal.

$HCF = 13, LCM = 182$

(ii) 510 and 92
Step 1: Prime factorization

$$510 = 2 \times 3 \times 5 \times 17$$
$$92 = 2^2 \times 23$$

Step 2: HCF = common factor(s) = 2 (only prime 2 in common, lowest power 2^1)
Step 3: LCM =

$$2^2 \times 3 \times 5 \times 17 \times 23$$
$$= 4 \times 3 \times 5 \times 17 \times 23$$
$$= 4 \times 3 \times 5 = 60 (\text{then } 60 \times 17 = 1020, 1020 \times 23 = 23460)$$
$$LCM = 23460$$

Step 4: Verification

$$HCF \times LCM = 2 \times 23460 = 46920$$
$$510 \times 92 = 46920$$

Equal.

$$\boxed{HCF = 2, LCM = 23460}$$

(iii) 336 and 54
Step 1: Prime factorization

$$336 = 2^4 \times 3 \times 7$$
$$54 = 2 \times 3^3$$

Step 2: HCF = common primes 2 and 3, lowest power:

$$2^1 \times 3^1 = 6$$

Step 3: LCM = highest powers:

$$2^4 \times 3^3 \times 7 = 16 \times 27 \times 7$$
$$16 \times 27 = 432, 432 \times 7 = 3024$$
$$LCM = 3024$$

Step 4: Verification

$$HCF \times LCM = 6 \times 3024 = 18144$$
$$336 \times 54 = 18144$$

Equal.

$$\boxed{HCF = 6, LCM = 3024}$$

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3. Find the LCM and HCF of the following integers by applying the prime factorisation method.
(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Solution 3:

(i) 12, 15 and 21
Step 1: Prime factorisation

$$12 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$21 = 3 \times 7$$

Step 2: HCF
Common prime = 3 (lowest power 3^1)

$$HCF = 3$$

Step 3: LCM

Take highest powers: $2^2, 3^1, 5^1, 7^1$

$$LCM = 2^2 \times 3 \times 5 \times 7$$
$$= 4 \times 3 \times 5 \times 7$$
$$4 \times 3 = 12,\ 12 \times 5 = 60,\ 60 \times 7 = 420$$
$$LCM = 420$$
$$\boxed{HCF = 3, LCM = 420}$$

(ii) 17, 23 and 29
Step 1: Prime factorisation

$$17 = 17$$
$$23 = 23$$
$$29 = 29$$

Step 2: HCF
No common prime ⇒

$$HCF = 1$$

Step 3: LCM
All are distinct primes:

$$LCM = 17 \times 23 \times 29$$

First: $17 \times 23 = 391$
Then $391 \times 29 = 11339$

$$LCM = 11339$$
$$\boxed{HCF = 1, LCM = 11339}$$

(iii) 8, 9 and 25
Step 1: Prime factorisation

$$8 = 2^3$$
$$9 = 3^2$$
$$25 = 5^2$$

Step 2: HCF
No common primes ⇒

$$HCF = 1$$

Step 3: LCM
Highest powers: $2^3, 3^2, 5^2$

$$LCM = 2^3 \times 3^2 \times 5^2$$
$$= 8 \times 9 \times 25$$
$$8 \times 9 = 72,\ 72 \times 25 = 1800$$
$$LCM = 1800$$
$$\boxed{HCF = 1, LCM = 1800}$$

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4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution 4: Given that HCF (306, 657) = 9, find LCM (306, 657)

We use the formula:

$$\textbf{HCF}(\boldsymbol{a}, \boldsymbol{b}) \times \textbf{LCM}(\boldsymbol{a}, \boldsymbol{b}) = \boldsymbol{a} \times \boldsymbol{b}$$

Given:

$$a = 306, b = 657, \text{HCF} = 9$$
$$9 \times \text{LCM} = 306 \times 657$$

First, compute 306×657:

$$306 \times 657 = 306 \times (600 + 57) = 306 \times 600 + 306 \times 57$$
$$306 \times 600 = 183,600$$
$$306 \times 57 = 306 \times (50 + 7) = 15300 + 2142 = 17442$$

Sum:

$$183600 + 17442 = 201,042$$

So:

$$9 \times \text{LCM} = 201,042$$
$$\text{LCM} = \frac{201042}{9} = 22338$$
$$\boxed{22338}$$

5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution 5: Check whether 6^n can end with the digit 0 for any natural number n

A number ends with digit 0 if it is divisible by both 2 and 5 (i.e., divisible by 10).

$$6^n = (2 \times 3)^n = 2^n \times 3^n$$

Prime factorization of 6^n contains only primes 2 and 3, no factor of 5.

Therefore 6^n is never divisible by 5, so it can never end with digit 0.

$$\boxed{\text{No}}$$

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Solution 6: (i) $7 \times 11 \times 13 + 13$

Take 13 common:

$$13 \times (7 \times 11 + 1) = 13 \times (77 + 1) = 13 \times 78$$

Since it is a product of two integers greater than 1 (13 and 78), it has factors other than 1 and itself ⇒ Composite.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

That's $7! + 5$.
Take 5 common:

$$5 \times \left(\frac{7!}{5} + 1\right) = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$$
$$\frac{7!}{5} = 1008$$

So:

$$5 \times (1008 + 1) = 5 \times 1009$$

Since 1009 > 1 and 5 > 1, product of two integers > 1 ⇒ Composite.

$$\boxed{\text{Both are composite}}$$

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the

Solution 7: Circular path problem
Sonia → 18 minutes per round
Ravi → 12 minutes per round
Start together at the same point and time.
They meet again at starting point at LCM of their times.

$$\text{LCM}(18, 12)$$
$$18 = 2 \times 3^2 12 = 2^2 \times 3$$

LCM: $2^2 \times 3^2 = 4 \times 9 = 36$ minutes.
Interpretation: They will meet again at starting point after 36 minutes.

$$\boxed{36 \text{ minutes}}$$

If you meant to write the whole problem and it was cut off, the answer to the usual "when will they meet at start" is 36 minutes.

EXERCISE 1.2

1. prove that $\sqrt{5}$ is irrational.

Sol: **Step 1: Assume the opposite**

Assume that $\sqrt{5}$ is rational.
Then we can write it in the form:

$$\sqrt{5} = \frac{a}{b}$$

where a and b are **coprime integers** (i.e., HCF$(a, b) = 1$) and $b \neq 0$.

Step 2: Square both sides

$$5 = \frac{a^2}{b^2}$$
$$a^2 = 5b^2$$

This means a^2 is divisible by 5.

Step 3: Apply Euclid's Lemma
If a prime p divides a^2, then p divides a.
Since 5 is prime and $5 \mid a^2$, we conclude $5 \mid a$.
Let $a = 5k$ for some integer k.

Step 4: Substitute back

$$(5k)^2 = 5b^2$$
$$25k^2 = 5b^2$$
$$5k^2 = b^2$$

This means b^2 is divisible by 5, so again $5 \mid b$ (since 5 is prime).

Step 5: Contradiction
We found $5 \mid a$ and $5 \mid b$, so a and b have a common factor 5.
But initially we assumed a and b are coprime.
This is a **contradiction**.
Hence, our assumption that $\sqrt{5}$ is rational is **false**.

$$\boxed{\sqrt{5} \text{ is irrational}}$$

2. We want to prove that $3 + 2\sqrt{5}$ is irrational.

Solution 2:

Step 1: Assume the opposite

Assume that $3 + 2\sqrt{5}$ is rational.

Let:

$$3 + 2\sqrt{5} = r$$

where r is a rational number.

Step 2: Rearrange the equation

$$2\sqrt{5} = r - 3$$
$$\sqrt{5} = \frac{r-3}{2}$$

Step 3: Analyze the result

Since r is rational, $r - 3$ is rational, and dividing by 2 keeps it rational.

So the equation $\sqrt{5} = \frac{r-3}{2}$ means $\sqrt{5}$ is rational (because the right-hand side is rational).

Step 4: Contradiction

We know $\sqrt{5}$ is irrational (proved earlier).
This is a contradiction.

Therefore, our assumption that $3 + 2\sqrt{5}$ is rational must be false.

Hence, $3 + 2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals:

(i). $\frac{1}{\sqrt{2}}$

Solution (i): Step 1: Assume the opposite

Assume $\frac{1}{\sqrt{2}}$ is rational.

So, we can write:

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

where a and b are integers, coprime (i.e., HCF$(a, b) = 1$), and $b \neq 0$.

Step 2: Rearrange

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$
$$\sqrt{2} = \frac{b}{a}$$

Step 3: Analyze

Since a and b are integers, $\frac{b}{a}$ is a rational number.
Thus, $\sqrt{2}$ would be rational.

Step 4: Contradiction

We know $\sqrt{2}$ is irrational (standard proof via contradiction).
This contradicts the assumption.

Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.

$$\boxed{\frac{1}{\sqrt{2}} \text{ is irrational}}$$

(ii). $7\sqrt{5}$

Solution (ii). Step 1: Assume the opposite
Assume $7\sqrt{5}$ is rational.

That means:

$$7\sqrt{5} = \frac{a}{b}$$

where a and b are coprime integers, $b \neq 0$.

Step 2: Rearrange

$$\sqrt{5} = \frac{a}{7b}$$

Step 3: Analyze
Since a and b are integers, $a/(7b)$ is rational.
So $\sqrt{5}$ is rational.

Step 4: Contradiction
We know $\sqrt{5}$ is irrational (proved earlier).
This is a contradiction.

Conclusion
Thus, $7\sqrt{5}$ must be irrational.

$$\boxed{7\sqrt{5} \text{ is irrational}}$$

(iii) $6 + \sqrt{2}$

Solution (iii): Step 1: Assume the opposite

Assume $6 + \sqrt{2}$ is rational.

Then we can write:

$$6 + \sqrt{2} = r$$

where r is a rational number.

Step 2: Rearrange

$$\sqrt{2} = r - 6$$

Step 3: Analyze

Since r and 6 are rational, $r - 6$ is rational.
So $\sqrt{2}$ would be rational.

Step 4: Contradiction

We know $\sqrt{2}$ is irrational (standard proof).
This contradicts the assumption.

Conclusion

Therefore, 6+ $\sqrt{2}$ is irrational.

$$\boxed{6 + \sqrt{2} \text{ is irrational}}$$